Interchange mode in the presence of dust

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The linear and nonlinear development of an electrostatic interchange mode which involves a magnetized nonuniform electron-ion fluid in the presence of nonuniform static charged dust grains is investigated. The charge on grains is taken as spatially dependent, and the consequences of that condition are investigated. It is shown that standardly accepted stabilization of the interchange mode in the presence of negatively charged grains can be violated due to the spatial dependence of the charge on grains. Also, the ion drift, which is caused by the action of a gravity term perpendicular to the magnetic field lines, is taken as nonuniform as a result of the magnetic field nonuniformity, and it is shown that due to such a nonuniformity the instability condition can be significantly modified. In the nonlinear regime several types of coherent stationary vortex structures are found: namely, dipolar and tripolar vortices and vortex chains. The dipolar vortex is found to propagate in the direction of the ion drift, while the tripole and vortex chains are carried by the drift flow. The spatial dependence of these structures is determined by parameters describing the nonuniformity of the equilibrium plasma.

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I. INTRODUCTION

The presence of dust in space plasmas has been the subject of many studies in the past several decades. Various effects of dust were discussed by Spitzer in 1941 [1] and in his renowned book [2]. Particularly interesting is Ref. [1] because it is such an early work on the matter of dusty plasmas. There, it is shown that for an electron density exceeding the value of 10^{-3} cm^{-3}, the charge on dust grains is determined by collisions with electrons rather than by the photoelectric effect, and the potential on grains is about -2 V. On the other hand, studies of wave propagation in the presence of dust started in 1985 [3], followed by many other studies, such as Refs. [4,5]. The presence of dust in a system introduces some new physical phenomena, such as extremely-low-frequency modes, charge fluctuation, crystal formation, etc. In the simplest case of a wave motion, the dust is subject to very-low-frequency sound-type oscillations [5] that are well separated from standard plasma modes and typical plasma frequencies.

The charge fluctuation on dust grains is another phenomenon which is absent in ordinary plasmas, but can be of great importance in a dusty plasma. It is typically a high-frequency process that can influence standard plasma modes. An early discussion of that effect can be found in the Spitzer's book [2], while in the theory of the propagation of waves in dusty plasmas the first studies can be found in Refs. [6,7]. Recently, there have been many studies dealing with the effect of fluctuating charge on dust grains, though one could conclude that there are tendencies of introducing that effect inappropriately in the problems where in fact it cannot be of any influence. An analysis of that issue is done in Ref. [8]; it is shown that the charge fluctuation is unimportant whenever wave frequencies are much smaller or much bigger than the typical charging frequency.

For very-low-frequency processes (e.g., the interchange or Rayleigh-Taylor mode) the charging and discharging of grains is an ''adiabatic'' process that is insignificant for the wave behavior. In fact, although the charge on dust grains in principle fluctuates (even in the absence of perturbations), for this frequency range the amount of charge on grains on average can be taken as constant. For some astrophysical conditions analytical estimates [9] show that the average charge on a grain, $\langle eZ_d\rangle$, can be given by

$$\langle eZ_d\rangle \approx -\frac{1}{1+(\tau_0/\tau)^{1/2}}+\psi\tau,$$

where $\tau=aT/e^2\leqslant 0.2$, T is the plasma temperature, a is the grain radius, the quantity τ_0 is the reduced temperature for which the ion collision rate with a negatively charged grain with $Z_d=-1$ equals the electron collision rate with a neutral grain, ψ is the solution to a transcedental Spitzer's equation [1,9], and its values for an electron-proton and a heavy-ion plasma are, respectively, -2.504 and -3.799. For $\tau\approx 0.1$, 10, 100, we have $\langle eZ_d\rangle \approx 1,30,300$, respectively. In the case of a magnetized plasma the motion of plasma particles and, consequently, the charging cross sections (i.e., sticking of electrons and ions on grains) are substantially changed; in the vicinity of a charged grain a magnetic bottle configuration is formed, etc. The cross sections are modified due to the image charge effect as well. More details on these effects, which are, however, not the subject of the present study, one can find in Refs. [9–11].

The situation is quite different when the wave frequency is comparable to the charge fluctuation frequency. Examples of such a situation can be found in Refs. [12,13], where these widely separated slow and fast times scales are discussed—namely, dust acoustic and charge fluctuation time scales,

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which emanate from the large inertia of the dust and from its collision with plasma particles, respectively. At fast time scales, it is the collective behavior of plasma particles (electrons and ions) which is affected by dust charge fluctuations. The existence of an ion temperature threshold above which the ion acoustic mode can become unstable due to the fluctuation of the dust charge is demonstrated. In Ref. [12] the effects of a fluctuating grain charge in the presence of a current-driven dust-ion-acoustic mode are investigated. The frequency and growth rate of the ion acoustic mode is shown to be clearly influenced by the fluctuating charge. Also, the existence of a dust charge fluctuation mode in the system is reported, and it is shown that the mode is unstable for the negative charge on dust per unit volume not exceeding some critical value.

The Rayleigh-Taylor (or the interchange) mode is well known from fluid dynamics and standard plasma theory. It appears when a heavy fluid is supported by a lighter one; the role of the lighter fluid in plasmas can be sometimes played by a magnetic field. The effective interchange instability develops also in situations when a nonuniform fluid (or two fluids with different densities) is (are) accelerated in the direction perpendicular to the density gradient (or perpendicular to the interface of the two fluids) [14]. A similar situation may appear in plasmas as well: as an example, in the dayside part of the magnetosphere which, after a compression by the solar wind, bounces back in the direction opposite to the gravity of planet. The instability is found to play an important role in the problems of accretion disks [15] and star forming clouds [16]. Knowledge about the latter is based on the recent development of observational tools (Hubble space telescope) which reveals the existence of various fingerlike (or elephant-trunk-like) structures in large clouds (like the Eagle Nebula). The stability analysis of the interfaces between such different media reveals the possibility for the Kelvin-Helmholtz, Rayleigh-Taylor, and Jeans instabilities, as starting points in the process of formation of stars.

In dusty plasmas, the presence of immobile negatively charged dust grains turns out to be stabilizing with respect to that mode, while the situation is opposite for positively charged grains [8]. The interchange mode that develops in the dust fluid itself has been studied recently in Refs. [17,18]; the studies were performed without the effect of the charge fluctuation. The effects of dust on planetesimal formation one can find in [19]. The Rayleigh-Taylor instability is shown to develop in the interaction of a shock wave with a presolar dusty nebula [20], where basically a dense fluid is accelerated into a less dense one, as mentioned in the text above. The instability appears first in the form of multiple clumps at the edge of the compressed material, which are afterwards driven inward in the form of fingers of the shock material that penetrates into the cloud.

In the present work we investigate the effects of a nonuniform magnetic field on the interchange mode in an electron-ion-dust plasma. This nonuniformity should be a natural feature of any space dusty plasma. We allow also for the grain charge to be spatially dependent which is another natural dusty plasma feature, which in return modifies the instability condition. The presence of some gravity g term and the nonuniform magnetic field results in a nonuniform drift of ions. That nonuniformity turns out to play an important role in both the linear and nonlinear regimes. In the linear regime it modifies the instability condition, while in the nonlinear regime it is responsible for the formation of a specific type of vortical structures consisting of monopolar and quadrupolar parts. The existence of dipolar vortices and vortex chains is also discussed. The analytical solutions and corresponding physical conditions allowing for such kind of solutions are presented in detail. In Sec. II we give the model and basic equations describing the most unstable, i.e., purely perpendicular, perturbations with respect to the magnetic field lines. In Sec. III we discuss the interchange instability in the presence of a spatial nonuniformity of both the magnetic field and the charge on dust grains. Particular nonlinear solutions are presented in Secs. IV–VI; a tripolar vortex (consisting of a monopolar and a quadrupolar part), found in Sec. IV, is shown to be driven by the aforesaid nonuniformities. A propagating dipole, presented in Sec. V, is a typical solution in the case of a constant ion drift. A vortex chain, found in Sec. VI, turns out to be strictly determined by the parameters describing the equilibrium nonuniformities and, similar to the tripole, it is carried by the nonuniform ion drift. At the end a summary is given.

II. MODEL AND DERIVATIONS

We start from a model which includes a nonuniform quasineutral plasma consisting of electrons, ions, and heavy dust grains, with density gradients of all plasma species along the x axis, so that in the equilibrium the following conditions is satisfied:

$$n_{i0}(x)=n_{e0}(x)+Z_d(x)n_{d0}(x). \tag{1}$$

Here $Z_d(x)$ denotes the charge residing on dust grains, which we take as spatially dependent. This should be a realistic situation for many space dusty plasma environments. As example, if the grains are charged due to the attachment of electrons and ions in the process of inelastic collisions, the spatial distribution of plasma particles will influence the average amount of charge on grains; in that case the gradients ∇Z_d and $\nabla n_{e0,i0}$ will have more or less the same direction. On the other hand, if the charge on grains is caused by the photoeffect, i.e., by an external source, it will depend on the distance from the source and the gradients can have opposite directions. Secondary emission due to energetic (external) plasma particles could depend on the density of the dust fluid, resulting again in oppositely oriented gradients. In our model Z_d can be positive (for negatively charged grains) and negative (in the opposite case).

For frequencies that are reasonably larger than the characteristic frequencies of the dust fluid, but less than the ion gyrofrequency,

$$\omega_{pd},\Omega_d \ll \omega \ll \Omega_i,$$

the grains can be assumed as heavy, i.e., stationary, taking no part in the motion of the perturbed fluid. Here ω_{pd},Ω_d are the dust Langmuir and dust gyrofrequencies, respectively.

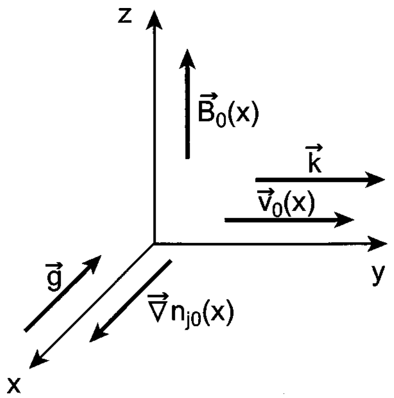

FIG. 1. The geometry of the problem.

The plasma is immersed in a nonuniform magnetic field oriented in the positive direction of the z axis, which we conveniently take in the form

$$\vec{B}_0(x)=B_T\frac{1}{\alpha(x)}\vec{e}_z. \tag{2}$$

Here, the term $\alpha(x)$ in general describes a magnetic field nonuniformity in the x direction. This form is convenient for the present analytical study because, as will be seen, B_0 enters appropriate expressions in the denominators, which will therefore result in the x-dependent term in the nominators. It allows also for separating the main part from the weak spatial dependence, which will be used in the calculations. The gravity term, which can be a real gravity or an effective one due to the plasma motion, is taken in the negative direction of the x axis, $\vec{g}=-g\vec{e}_x$, where $g=\text{const}>0$. Density gradients are in the direction of the x axis, $\vec{\nabla} n_{j0}=n'_{j0}(x)\vec{e}_x$, where $n'_{j0}(x)>0$ at least for electrons and ions, which take part in perturbations. The prime here and in the rest of the text denotes a derivative in the x direction, and j stays for e,i,d.

Thus we have a geometry, presented in Fig. 1, which allows for the interchange (or Rayleigh-Taylor) instability; it is chosen in the form corresponding to Ref. [8]. Note that even for necessarily growing densities n_{i0}, n_{e0} with x, the product $Z_d(x)n_{d0}(x)$ in principle yields various possibilities.

The momentum equation for plasma particles is written in the form

$$m_jn_j\left(\frac{\partial}{\partial t}+\vec{v}_j\vec{\nabla}\right)\vec{v}_j=q_jn_j(-\vec{\nabla}\phi+\vec{v}_j\times\vec{B}_0)+m_jn_j\vec{g}-\nabla p_j. \tag{3}$$

In the limit of inertialess electrons, from Eq. (3) it follows that in the equilibrium they will be subject to a drift in the negative y direction [for a positive $\alpha(x)$], given by

$$\vec{v}_{e0}(x)=\frac{T_{e0}}{en_{e0}B_0}\vec{e}_z\times\vec{\nabla} n_{e0}=-\alpha(x)\frac{c_s^2}{\Omega_T}\frac{n'_{e0}}{n_{e0}}\vec{e}_y,$$
$$c_s^2=\frac{T_{e0}}{m_i},\quad \Omega_T=\frac{eB_T}{m_i}. \tag{4}$$

Similarly, for ions we have a positive drift given by

$$\vec{v}_{i0}(x)=\left(\frac{g}{\Omega_T}+\frac{c_i^2}{\Omega_T}\frac{n'_{i0}}{n_{i0}}\right)\alpha(x)\vec{e}_y,\quad c_i^2=\frac{T_{i0}}{m_i}. \tag{5}$$

For a convenience which will be obvious in the following text, we shall write the ion equilibrium drift in the form

$$\vec{v}_{i0}(x)=\frac{1}{B_T}\varphi'(x)\vec{e}_y=\frac{1}{B_T}\vec{e}_z\times\vec{\nabla}\varphi, \tag{6}$$

where $\varphi(x)$ is a stream (drift) function or an effective equilibrium potential caused by the g term.

For the most unstable case, i.e., for electrostatic perturbations propagating perpendicular to the magnetic field lines, the electron motion is described approximately by

$$\vec{v}_{\perp e1}=\frac{1}{B_0(x)}\vec{e}_z\times\vec{\nabla}\phi-\frac{c_s^2}{\Omega_0}\vec{e}_z\times\frac{\vec{\nabla} n_{e1}}{n_{e0}},\quad \Omega_0=\frac{eB_0}{m_i}. \tag{7}$$

The parallel particle motion and electromagnetic effects have been discussed elsewhere (see Refs. [21,22]).

We use also the continuity equation for electrons,

$$\left[\frac{\partial}{\partial t}+(\vec{v}_{e0}+\vec{v}_{e1})\cdot\vec{\nabla}\right](n_{e0}+n_{e1})+n_{e0}\vec{\nabla}\cdot\vec{v}_{e1}=0. \tag{8}$$

A similar set of equations is written for ions:

$$\vec{v}_{\perp i1}=\frac{1}{B_0(x)}\vec{e}_z\times\vec{\nabla}\phi+\frac{c_i^2}{\Omega_0}\vec{e}_z\times\frac{\vec{\nabla} n_{i1}}{n_{i0}}$$
$$-\frac{1}{\Omega_TB_T}\left[\frac{\partial}{\partial t}+\frac{1}{B_T}\vec{e}_z\times\vec{\nabla}(\phi+\varphi)\cdot\vec{\nabla}\right]\vec{\nabla}(\phi+\varphi), \tag{9}$$

$$\left[\frac{\partial}{\partial t}+(\vec{v}_{i0}+\vec{v}_{i1})\cdot\vec{\nabla}\right](n_{i0}+n_{i1})+n_{i0}\vec{\nabla}\cdot\vec{v}_{i1}=0. \tag{10}$$

In writing the above expressions the equilibrium gradients are assumed as the first-order terms, and we keep linear and nonlinear small terms up to second order.

III. LINEAR INSTABILITY

Linearized equations (7)–(10) for perturbations of the form $\hat{\zeta}(x)\exp(-i\omega t+iky)$, where $\hat{\zeta}(x)$ is the x-dependent wave amplitude, with the help of the quasineutrality condition for the static dust grains

$$n_{e1}=n_{i1}, \tag{11}$$

yield the following quasinonlocal equation for the potential amplitude:

$$\left[\frac{\partial^2}{\partial x^2}-k^2-\frac{k\Omega_T B_T}{n_{i0}(\omega-kv_0)}\left(\frac{1}{B_0}\right)'\left(Z_d n_{d0}+\frac{v_0 k n_{e0}}{\omega}\right)\right.$$
$$\left.-\frac{k\Omega_T}{n_{i0}}\frac{(Z_d n_{d0})'}{\omega-kv_0}-\frac{v_0 k^2\Omega_T}{\omega(\omega-kv_0)}\frac{n_{e0}'}{n_{i0}}+\frac{kv_0''}{\omega-kv_0}\right]\hat{\phi}(x)$$
$$=0. \qquad (12)$$

Although Eq. (12) comprises the shear flow (drift) terms resembling the Kelvin-Helmholtz (or Rayleigh) instability, one should note that these terms vanish in the absence of the g term. Therefore, the shear flow effects here cannot be studied separately from the g effects. This is a principal difference in comparison with some classical works dealing with shear flow effects on the interchange instability [23]. Rather, one should discuss the modification of the Rayleigh-Taylor (the interchange) instability due to the nonuniformity effects. Note the terms in parentheses, multiplying the term $(1/B_0)'$; the first one is due to the presence of dust while the second is the g term caused by the difference in the ion and electron masses.

Equation (12) is nontrivial to solve in general but it can be analyzed in various limits and for some specific given equilibrium profiles for $Z_d(x), n_{d0}(x), n_{e0}(x), n_{i0}(x), v_0(x)$. Note that in deriving Eq. (12) the thermal corrections have been omitted because they introduce only minor modifications of the instability conditions, as has been shown in Ref. [8].

For the uniform magnetic field, and therefore constant drift v_0, Eq. (12) yields the following dispersion equation:

$$\omega^2+\omega\left[\frac{\Omega_T}{k}\frac{[Z_d(x)n_{d0}(x)]'}{n_{i0}}-v_0 k\right]+v_0\Omega_T\frac{n_{e0}'}{n_{i0}}=0. \qquad (13)$$

One can follow the assumptions from Ref. [8] and take the exponential densities

$$n_{i0}(x)=N_{i0}\exp(\lambda x),\quad n_{e0}(x)=N_{e0}\exp(\lambda x), \qquad (14)$$

in the case when

$$n_{d0}(x)/n_{i0}(x)=\epsilon=\text{const}. \qquad (15)$$

Note that this is equivalent to the case of a spatially constant charge Z_d, which is easily seen by taking the x derivative of the quasineutrality condition (1) and using Eqs. (14) and (15).

Now, in the ion reference frame from Eq. (13), using

$$\frac{n_{e0}'}{n_{i0}}=\left(\frac{n_{e0}}{n_{i0}}\right)'+\frac{n_{e0}}{n_{i0}}\frac{n_{i0}'}{n_{i0}}=(1-\epsilon Z_d)\lambda$$

and introducing $\Omega=\omega-v_0 k$, we obtain

$$k\Omega^2+\Omega(v_0k^2+\Omega_T\epsilon\lambda Z_d)+v_0\Omega_T\lambda k=0,\quad v_0=\frac{g}{\Omega_T}. \qquad (16)$$

Equation (16) is identical to the dispersion equation derived in Ref. [8]. From the instability condition which follows from Eq. (16),

$$\lambda g>\frac{1}{4}\left(\frac{\epsilon\lambda\Omega_T Z_d}{k}+kv_0\right)^2, \qquad (17)$$

it can be seen that, in the frame of the model described by Eqs. (14) and (15), the presence of negatively charged dust (positive Z_d) stabilizes the system by raising the threshold for λg above which, for a given k, the system is unstable. The situation is opposite for positively charged grains, and these results are known from Ref. [8]. Without dust it yields the standard classical instability condition.

However, for a spatially nonuniform charge on grains, from Eq. (13) one obtains the instability condition for arbitrary density profiles (with positive gradients) in the form

$$g\frac{n_{i0}'}{n_{i0}}>\frac{1}{4}\left[\frac{\Omega_T}{k}\frac{Z_d'(x)n_{d0}(x)+Z_d(x)n_{d0}'(x)}{n_{i0}(x)}+v_0k\right]^2. \qquad (18)$$

Here we have taken $n_{e0}'=(n_{i0}-Z_d n_{d0})'$ in the last term in Eq. (13). Obviously the spatial dependence of $Z_d(x)$ can in principle make the threshold lower compared to Eq. (17), so that the mode is destabilized even in the presence of negatively charged grains. From physical reasons this is possible only when the charge on grains is not mainly caused by the absorption of plasma particles; instead it should be due to some other physical mechanisms which cause the derivative of Z_d to have a negative sign, with respect to the density gradients. Note also that, due to the same reason, the phase velocity of the mode

$$\frac{\omega_r}{k}=\frac{v_0}{2}-\frac{\Omega_T}{2k^2}\frac{[Z_d(x)n_{d0}(x)]'}{n_{i0}(x)}, \qquad (19)$$

where ω_r is the real frequency, is not necessarily decreased in the presence of negatively charged grains. Evidently, the dispersion of the mode is due to the presence of dust.

For the case of a linear $\alpha(x)$ and in the local limit $\partial/\partial x \approx k_x^2 \ll k^2$, from Eq. (12) one can derive the following dispersion equation:

$$\omega^2+\left[\frac{\Omega_T Z_d n_{d0}\alpha'}{n_{i0}k}+\frac{\Omega_T(Z_d n_{d0})'}{n_{i0}k}-v_0k\right]\omega+\frac{v_0 n_{e0}\Omega_T\alpha'}{n_{i0}}$$
$$+\frac{v_0\Omega_T n_{e0}'}{n_{i0}}=0, \qquad (20)$$

where $v_0\equiv g/\Omega_T$. The instability condition can be written in the form

$$\frac{1}{4}\left[\frac{\Omega_T Z_d n_{d0}\alpha'}{n_{i0}k}+\frac{\Omega_T(Z_d n_{d0})'}{n_{i0}k}+v_0k\right]^2<g\left(\frac{n_{i0}'}{n_{i0}}+\alpha'\right). \qquad (21)$$

Consequently, the instability condition can be substantially modified due to the nonuniformity of the ion drift. The modi-

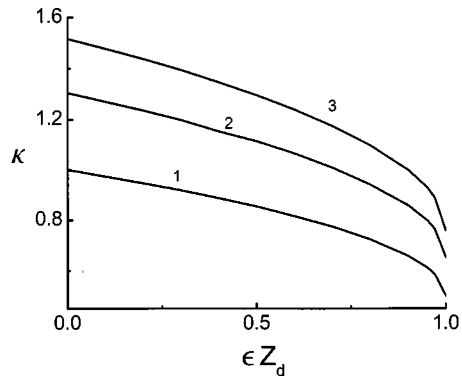

FIG. 2. The threshold values of κ (in units of k_*) versus ϵZ_d. Here curves 1, 2, and 3 correspond to $\alpha'=0$, $\alpha'=0.7$, and $\alpha'=1.3$, respectively. The values of κ above the corresponding curves are stable for the given model.

fication is twofold. First, as it enters the left-hand side of Eq. (21), for a positive gradient α' in principle it raises the threshold for the unstable mode similar to the effect of dust; though one should note that the dust and shear drift terms are coupled and various possibilities can take place. Second, the mode may become unstable even for a constant ion density provided that the perpendicular drift gradient is bigger that the critical value which follows from Eq. (21). This is to some extent similar to a classical result dealing with the streaming instability in a plasma [24]; note, however, that here the instability is strictly related to the presence of the g term.

A direct comparison with Ref. [8], regarding the spatial dependence of Z_d introduced here, cannot be done since the results in Ref. [8] are obtained for a specific case of density distribution, given by Eqs. (14) and (15), which is equivalent to a constant Z_d. However, the effects of the nonuniform drift studied here can be compared with the results of Ref. [8]. For the profiles (14) and (15), the instability condition (21) can be rewritten as

$$\left(\frac{\epsilon Z_d \alpha'}{\lambda}+\epsilon Z_d+4\kappa^2\right)^2<16\kappa^2\left(1+\frac{\alpha'}{\lambda}\right). \tag{22}$$

Here we introduce the wave number κ, normalized to $k_*=2\Omega_T\sqrt{\lambda/g}$. For $\alpha'=0$, Eq. (22) is identical to the corresponding one from Ref. [8] for the negative charge on grains. Now, one can calculate the unstable values of κ:

$$\kappa<\left\{\frac{1}{4}\left(1+\frac{\alpha'}{\lambda}\right)(2-\epsilon Z_d)+\frac{1}{2}\left[\left(1+\frac{\alpha'}{\lambda}\right)^2(1-\epsilon Z_d)\right]^{1/2}\right\}^{1/2}. \tag{23}$$

In the case $\alpha'=0$, Eq. (23) yields the corresponding result from Ref. [8] for the negative charge on grains. In the absence of dust it yields the well-known result $k^2/4\Omega_T^2<\lambda/g$. The threshold values of κ versus ϵZ_d are presented in Fig. 2, for two values of the drift gradient α' in units of λ, i.e., for $\alpha'=0.7$ (curve 2), and $\alpha'=1.3$ (curve 3). Curve 1 corresponds to the one from Ref. [8] (the case $\alpha'=0$). Here ϵZ_d takes values from ≈ 0 (a negligible presence of dust in the plasma) to ≈ 1 (all electrons attached to the grains). The values κ above the curves are stable for the given model.

Note a very important consequence of the drift nonuniformity which follows from Eq. (21) or (22); namely, in the limit $\alpha'+n_{i0}'/n_{i0}<0$ the instability vanishes. That happens whenever the decreasing drift changes faster than the logarithm of the increasing ion density, and this conclusion is valid generally, not only for a dusty plasma.

IV. NONLINEAR QUADRUPOLAR VORTEX

Using Eqs. (7) and (9), the nonlinear continuity equations (8) and (10), without thermal corrections, can be written as

$$\left(\frac{\partial}{\partial t}+\frac{1}{B_T}\vec{e}_z\times\vec{\nabla}\phi\cdot\vec{\nabla}\right)\left[\ln n_{e0}(x)+n_{e1}+\frac{B_T}{B_0(x)}\right]=0, \tag{24}$$

$$\left[\frac{\partial}{\partial t}+\frac{1}{B_T}\vec{e}_z\times\vec{\nabla}(\phi+\varphi)\cdot\vec{\nabla}\right]\left[\ln n_{i0}(x)+n_{i1}+\frac{B_T}{B_0(x)}-\frac{1}{\Omega_T B_T}\nabla^2[\phi+\varphi(x)]\right]=0. \tag{25}$$

Here the perturbed concentrations n_{e1}, n_{i1} are normalized to n_{e0}, n_{i0}. The nonlinearity in the above equations is of the vector-product (or Poisson bracket) type, implying vorticity due to the leading order $\vec{E}\times\vec{B}$ drift. Directly comparing typical linear and nonlinear terms in the corresponding equations one can conclude in what situations the nonlinear terms are of importance. Hence, in Eq. (25) we make the ratio of the following typical terms: $\epsilon_1=\vec{e}_z\times\vec{\nabla}\phi\cdot\vec{\nabla}\ln n_{i0}(x)/B_T$ and $\epsilon_2=\vec{e}_z\times\vec{\nabla}\phi\cdot\vec{\nabla}\nabla^2\phi/\Omega_T B_T^2$. This yields $\epsilon_2/\epsilon_1=kL_{n0}\,k^2\phi/\Omega_T B_T$. Note that the term kL_{n0} is (much) bigger than 1 as assumed throughout the text; therefore the terms $\epsilon_{1,2}$ can be of the same order even if the perturbation ϕ is very small (i.e., much less than the term $\Omega_T B_T/k^2$, which is, however, dependent on the wave number k). In the literature it is known that such nonlinear terms can cause the formation of quasistationary vortical structures that can propagate in the system. We shall therefore assume the existence of such nonlinear solutions that can develop in the process of growing the unstable mode and search for their analytical description and for the physical conditions that make such solutions possible. Consequently, we propose traveling solutions that either can be carried by the drift v_0 or can propagate in the system independently with the velocity u_y in the direction of propagation of the linear mode. Writing

$$\frac{\partial}{\partial t}=-u_y\frac{\partial}{\partial y}=-\vec{e}_z\times\vec{\nabla}u_y x,$$

Eqs. (24) and (25) can be written in a proper vector-product form which allows for an integration, yielding the following expressions:

$$\ln n_{e0}(x)+n_{e1}+\frac{B_T}{B_0(x)}=f_1(\phi-B_T u_y x), \tag{26}$$

$$\ln n_{i0}(x)+n_{i1}+\frac{B_T}{B_0(x)}-\frac{1}{\Omega_T B_T}\nabla^2[\phi+\varphi(x)]$$
$$=f_2(\phi+\varphi-B_T u_y x). \tag{27}$$

The expressions (26) and (27) are very general as $f_1(\xi_1)$, $f_2(\xi_2)$ are arbitrary, i.e., any functions of the given arguments. In order to find some specific, particular nonlinear solutions one has to specify the form of these two functional forms. Thus, we proceed by taking a particular, in principle piecewise linear shape of the functions as

$$f_1(\xi_1)=f_1\cdot\xi_1, \quad f_2(\xi_2)=f_2\cdot\xi_2, \tag{28}$$

where $f_{1,2}$ are some constants which we allow to have two different values in space. Namely, we divide the space by a circle of radius r_0 and allow for different values of these constants outside and inside the circle, in the following text denoted as $f_{1,2}^{out,in}$. By such a choice we keep nonlinearity in its simplest form, as that means that the functional forms $f_1(\xi_1), f_2(\xi_2)$ are in fact nonlinear.

In that case from Eq. (26) one can obtain

$$n_{e1}=f_1\cdot\phi, \tag{29}$$

where, in order to have localized solutions for $n_{e1}(r,\theta)$ and $\phi(r,\theta)$, the following condition must be satisfied:

$$f_1 B_T u_y x+\ln n_{e0}(x)+\frac{B_T}{B_0(x)}=0. \tag{30}$$

Using the quasineutrality condition (11) we combine Eqs. (26) and (27) and obtain the following equation for the perturbed nonlinear potential:

$$(\nabla^2+F_2-F_1)\phi+\varphi''(x)+F_2\varphi(x)-(F_2-F_1)B_T u_y x$$
$$-\Omega_T B_T \ln\left[\frac{n_{i0}(x)}{n_{e0}(x)}\right]=0, \tag{31}$$

where $F_{1,2}=f_{1,2}\Omega_T B_T$. We shall solve Eq. (31) independently outside and inside of the mentioned circle, and match the solutions smoothly at $r=r_0$. From the requirement of localized solutions, it is seen that when in the outside region the following condition is satisfied,

$$\varphi''(x)+F_2^{out}\varphi(x)-(F_2^{out}-F_1)B_T u_y x-\Omega_T B_T \ln\left[\frac{n_{i0}(x)}{n_{e0}(x)}\right]$$
$$=0, \tag{32}$$

from Eq. (31) we obtain

$$(\nabla^2-\lambda_1^2)\phi^{out}=0. \tag{33}$$

Here we have introduced the notation

$$-\lambda_1^2=F_2^{out}-F_1,$$

where, in view of the condition (30) which involves the continuous equilibrium functions, one has to take F_1 as constant in all space. However, the nonlinearity of $f_2(\xi_2)$ still holds.

A particular solution of Eq. (33), in terms of cylindrical coordinates r,θ, can be written as

$$\phi^{out}(r,\theta)=\alpha_0 K_0(\lambda_1 r)+\alpha_2 K_2(\lambda_1 r)\cos 2\theta. \tag{34}$$

Here $K_{0,1}$ are modified Bessel functions of the second kind. On the other hand, inside the circle we take

$$\varphi''(x)+F_2^{in}\varphi(x)-(F_2^{in}-F_1)B_T u_y x-\Omega_T B_T \ln\left[\frac{n_{i0}(x)}{n_{e0}(x)}\right]$$
$$=\kappa x^2, \tag{35}$$

where, in principle, κ is an arbitrary constant. The reason for taking the right-hand side of Eq. (35) in the given form is the following. When we combine Eqs. (32) and (35), the logarithm terms cancel out, and the simplest form of the nonuniformity which satisfies the resulting equation is such that it yields a linear spatial profile for the velocity $v_0(x)$, which corresponds to the quadratic profile of $\varphi(x)$. We therefore obtain

$$\varphi(x)=\frac{(\lambda_1^2+\lambda_2^2)u_y B_T}{F_2^{in}-F_2^{out}}x+\frac{\kappa}{F_2^{in}-F_2^{out}}x^2, \tag{36}$$

i.e.,

$$v_0(x)\equiv\frac{g}{\Omega_T}\alpha(x)=\frac{(\lambda_1^2+\lambda_2^2)u_y}{F_2^{in}-F_2^{out}}\left[1+\frac{2\kappa}{u_y B_T(\lambda_1^2+\lambda_2^2)}x\right]. \tag{37}$$

Here we have introduced

$$\lambda_2^2=F_2^{in}-F_1.$$

One should note that from $x^2=r^2(1+\cos 2\theta)/2$ even such a simple nonuniform case involves quadrupolar harmonics as possible solutions. Further, in Eq. (37) we can choose $g/\Omega_T=(\lambda_1^2+\lambda_2^2)u_y/(F_2^{in}-F_2^{out})$ and $\alpha(x)=1+2\kappa x/[u_y B_T(\lambda_1^2+\lambda_2^2)]$.

In this notation and on condition (35), from Eq. (31) we obtain

$$(\nabla^2+\lambda_2^2)\left(\phi^{in}+\frac{\kappa}{\lambda_2^2}x^2-\frac{2\kappa}{\lambda_2^4}\right)=0. \tag{38}$$

From the condition (30) and for an increasing electron density profile, as assumed in the model, we find out that both F_1 and κ must be negative, so we write them formally as $F_1=-b^2$, $\kappa=-\kappa_1^2$. Now we find the necessary electron density distribution

$$n_{e0}(x)=\frac{1}{e}\exp\left[\left(\frac{b^2 u_y}{\Omega_T}+\frac{2\kappa_1^2}{B_T(\lambda_1^2+\lambda_2^2)u_y}\right)x\right]. \tag{39}$$

Note that here e is the base of the natural logarithm.

In the same notation the solution of Eq. (38) can be written as

$$\phi^{in}(r,\theta)=-\frac{2\kappa_1^2}{\lambda_2^4}+\frac{\kappa_1^2}{2\lambda_2^2}r^2+\beta_0 J_0(\lambda_2 r)+\left[\beta_2 J_2(\lambda_2 r)+\frac{\kappa_1^2}{2\lambda_2^2}r^2\right]\cos 2\theta. \tag{40}$$

Here $J_{0,2}$ are Bessel functions of the first kind.

The imposed conditions (32) and (35) yield the corresponding equilibrium profiles for the ion drift velocity and the dust concentration, which must be satisfied in order to have solutions (34) and (40). Using $F_2^{in}=\lambda_2^2-b^2$ and $F_2^{out}=-(b^2+\lambda_1^2)$, we find

$$v_0(x)=u_y-\frac{2\kappa_1^2}{B_T(\lambda_1^2+\lambda_2^2)u_y}x. \tag{41}$$

It is seen that the proposed traveling (with the velocity u_y) solution is in fact carried by the ion drift whose amplitude is $g/\Omega_T=u_y$. Note also that the vortex appears for a decreasing profile of the drift velocity. It does not appear in the case of a uniform drift. From Eqs. (32) and (35) we also find

$$\frac{n_{i0}(x)}{n_{e0}(x)}\equiv 1+\frac{Z_d(x)n_{d0}(x)}{n_{e0}(x)}=\exp(a_1x^2+a_2x+a_3), \tag{42}$$

where

$$a_1=\frac{b^2+\lambda_1^2}{\Omega_T B_T}\quad\frac{\kappa_1^2}{\lambda_1^2+\lambda_2^2},\quad a_2=-\frac{u_y b^2}{\Omega_T},$$

$$a_3=-\frac{2\kappa_1^2}{\Omega_T B_T(\lambda_1^2+\lambda_2^2)}.$$

In the framework of the given model in the absence of dust the present solution does not appear, as can be seen from the quasineutrality condition $n_{i0}(x)=n_{e0}(x)$ and Eq. (42). However, a quadrupolar solution is possible in an electron-ion plasma as well; in that case the functional forms (28) should be taken as constant in the outside region.

The integration constants b, $\lambda_{1,2}$, $\alpha_{0,2}$, and $\beta_{0,2}$ and the physical parameters κ_1, r_0, and u_y should be found from appropriate physical conditions at the circle $r=r_0$.

The continuity of $\phi(r_0,\theta)$ yields

$$\alpha_0 K_0(\lambda_1 r_0)=-\frac{2\kappa_1^2}{\lambda_2^4}+\frac{\kappa_1^2 r_0^2}{2\lambda_2^2}+\beta_0 J_0(\lambda_2 r_0), \tag{43}$$

$$\alpha_2 K_2(\lambda_1 r_0)=\frac{\kappa_1^2 r_0^2}{2\lambda_2^2}+\beta_2 J_2(\lambda_2 r_0). \tag{44}$$

The continuity of $\vec{\nabla}\phi(r_0,\theta)$ yields

$$-\lambda_1\alpha_0 K_1(\lambda_1 r_0)=\frac{\kappa_1^2 r_0}{\lambda_2^2}-\beta_0\lambda_2 J_1(\lambda_2 r_0), \tag{45}$$

$$-\frac{\lambda_1\alpha_2}{2}[K_1(\lambda_1 r_0)+K_3(\lambda_1 r_0)]=\frac{\kappa_1^2 r_0}{\lambda_2^2}+\lambda_2\beta_2\left[J_1(\lambda_2 r_0)-\frac{2}{\lambda_2 r_0}J_2(\lambda_2 r_0)\right]. \tag{46}$$

The functional form $f_2(\xi_2)$ must be continuous as it is obtained after one integration. This implies the continuity of $\nabla^2\phi(r_0,\theta)$ as well. Formally this condition is written as

$$F_2^{out}\cdot[\phi^{out}+\varphi(x)-B_T u_y x]=F_2^{in}\cdot[\phi^{in}+\varphi(x)-B_T u_y x],$$

where we use Eq. (36) and separate the zeroth and second harmonics. For the zeroth harmonics this yields

$$\alpha_0 K_0(\lambda_1 r_0)=\frac{\kappa_1^2 r_0^2}{2(\lambda_1^2+\lambda_2^2)} \tag{47}$$

and, for the second,

$$\alpha_2 K_2(\lambda_1 r_0)=\frac{\kappa_1^2 r_0^2}{2(\lambda_1^2+\lambda_2^2)}. \tag{48}$$

The meaning of these two conditions is that the circle r_0 is an isoline for the function $\xi_2(r,\theta)\equiv\phi(r,\theta)+\varphi(x)-B_T u_y x$, i.e., $\xi_2(r_0,\theta)=0$.

Thus one integration constant and the physical parameters will remain free, which results in a broad spectrum of possibilities for the tripolar vortex to appear in the given system.

The solution presented by Eqs. (34) and (40) consists of the monopolar and quadrupolar parts. The contour plot of such a solution is known from the literature and turns out to have a tripolar form [25] consisting of a vortex core and two lateral vortices with opposite vorticity. The present structure is elongated along the magnetic field lines; it moves together with the ion drift, and its amplitude [see Eq. (40)] is strictly dependent on the gradient of the ion drift.

V. DIPOLAR VORTEX

In the case of a negligible nonuniformity of the magnetic field [i.e., for a constant velocity v_0 and a linear x-dependent $\varphi(x)$], instead of the x^2 term in the condition (35) we take κx, which is then used to cancel the growing terms at $x\to\infty$. On the other hand, such a term in cylindrical coordinates imposes the existence of first harmonics in the solution, i.e., $x=r\cos\theta$. Using Eq. (29) and $F_1=-b^2$, from Eq. (31) on conditions (30), (32), and (35) (where now instead of κx^2 we have κx) we find a particular solution in the form of a dipolar vortex

$$\phi^{out}(r,\theta)=c_0K_0(\lambda_1 r)+c_1K_1(\lambda_1 r)\cos\theta,\quad r>r_0, \tag{49}$$

$$\phi^{in}(r,\theta)=d_0J_0(\lambda_2 r)+\left[d_1J_1(\lambda_2 r)-\frac{\kappa}{\lambda_2^2}\,r\right]\cos\theta,\quad r<r_0, \tag{50}$$

and the concentration n_{e1} is again given by Eq. (29). Here $\lambda_{1,2}$ are the same quantities as in Sec. IV, and $c_{0,1}$ and $d_{0,1}$ are the new integration constants. In accordance with the assumed model, which includes the increasing densities in the positive x direction, from Eqs. (30), (32), and (35) we find as earlier that κ must be negative $(=-\kappa_1^2)$ and the electron and ion densities are given by

$$n_{e0}(x)=\exp\left(\frac{b^2u_y}{\Omega_T}x\right),$$

$$n_{i0}(x)=n_{e0}(x)\exp(cx)=\exp\left[\frac{\kappa_1^2(b^2+\lambda_1^2)}{\Omega_T B_T(\lambda_1^2+\lambda_2^2)}x\right], \quad (51)$$

where

$$c=\frac{\kappa_1^2(b^2+\lambda_1^2)}{\Omega_T B_T(\lambda_1^2+\lambda_2^2)}-\frac{b^2u_y}{\Omega_T}.$$

From the same conditions we find that the vortex velocity and the ion drift are related by

$$u_y=v_0+\frac{\kappa_1^2}{B_T(\lambda_1^2+\lambda_2^2)}. \quad (52)$$

Thus, the appropriate concentrations increase with x as required, the vortex speed is larger than the ion drift, as is known from standard vortex theory, and the parameter κ_1 describes the difference between these two velocities.

The dipolar vortex exists in the absence of dust as well. In that case from the requirement of quasineutrality this must be $c=0$, and we find

$$\kappa_1^2=u_yB_Tb^2\frac{\lambda_1^2+\lambda_2^2}{b^2+\lambda_1^2}$$

and

$$u_y=v_0\left(1+\frac{b^2}{\lambda_1^2}\right).$$

It is seen that in both cases, with and without dust, the vortex velocity is larger than the constant ion drift velocity v_0. The integration constants $c_{0,1}$ and $d_{0,1}$ can be found from boundary conditions similar to those in Sec. IV.

VI. VORTEX CHAIN

Localized solutions in the direction of equilibrium gradients, and periodic in the direction of the ion drift, can be obtained from Eqs. (26) and (27) by the following procedure. We use Eq. (29) for a constant f_1 in all space, but in Eq. (27) we take a highly nonlinear arbitrary function $f_2(\xi_2)$ in the form

$$f_2(\xi_2)=f_1\cdot\xi_2-\frac{1}{\Omega_T B_T}\frac{4c_1k_1^2}{d_1^2}\exp\left(-\frac{2}{c_1}\xi_2\right), \quad (53)$$

$$\xi_2=\phi+\varphi-B_Tu_yx.$$

Here c_1, k_1, and d_1 are some integration constants, and, as earlier, we search for solutions that propagate with the velocity u_y in the direction of the ion drift. Note that condition (30) still holds.

On the condition

$$\ln n_{i0}+\frac{B_T}{B_0}=f_1\cdot(\varphi-B_Tu_yx). \quad (54)$$

Equation (27) can be written as

$$\nabla^2(\phi+\varphi)=\frac{4c_1k_1^2}{d_1^2}\exp\left[-\frac{2}{c_1}(\phi+\varphi-B_Tu_yx)\right]. \quad (55)$$

One solution of Eq. (55) can be readily written in the form

$$\phi(x,y)=-\varphi(x)+B_Tu_yx+c_1\ln\left[2\left(\cosh k_1x+\sqrt{1-\frac{1}{d_1^2}}\cos k_1y\right)\right]. \quad (56)$$

Note that

$$\lim_{x\to\pm\infty}\ln[2\cosh(k_1x)]=k_1|x|,$$

where $|x|$ denotes the absolute value. Without $\varphi(x)$ the solution (56) represents the well-known Kelvin-Stuart-type vortex chain which is periodic in y, but physically inappropriate as it is not localized in the x direction. In the present case it can be localized in the x direction for the drift function $\varphi(x)$ satisfying

$$\lim_{x\to\pm\infty}\varphi(x)=(c_1k_1\pm B_Tu_y)|x|. \quad (57)$$

Here $d_1^2\geqslant1$; for the case $d_1=1$ periodicity in y vanishes and Eq. (56) transforms into a zonal flow.

Some of the integration constants in Eqs. (55) and (56) will be determined by the magnetic field nonuniformity which should be given. Indeed, let us take a particular case when the magnetic field (2) changes smoothly as

$$B_0(x)=\frac{B_T}{1-a^2\tanh\lambda_0x}$$

and, therefore,

$$v_0(x)=\frac{g}{\Omega_T}(1-a^2\tanh\lambda_0x), \quad (58)$$

where from physical reasons one might take $a^2<1$. From the condition (30) we find the appropriate profile for the electron density

$$n_{e0}(x)=\frac{1}{e}\exp(a^2\tanh\lambda_0 x+b_1^2B_Tu_yx). \tag{59}$$

Here, in order to have an increasing profile for $n_{e0}(x)$ we have taken $f_1=-b_1^2$. Using the definition of the drift function $\varphi(x)$ we find

$$\varphi(x)=B_T\int v_0(x)dx+c_0$$
$$=\frac{gB_T}{\Omega_T}x-\frac{ga^2B_T}{\Omega_T\lambda_0}\ln[2(\cosh\lambda_0x)]. \tag{60}$$

Note here that the integration constant is taken conveniently as $c_0=(\ln 2)/\lambda_0$. Now from Eq. (54) we can find the appropriate ion concentration

$$n_{i0}(x)=\frac{1}{e}\exp\left[a^2\tanh(\lambda_0x)+\frac{gb_1^2a^2B_T}{\lambda_0\Omega_T}\ln[2\cosh(\lambda_0x)]+b_1^2B_T\left(u_y-\frac{g}{\Omega_T}\right)\right]x. \tag{61}$$

From Eq. (60) we see that

$$\lim_{x\to\pm\infty}\varphi(x)=\left(\pm\frac{gB_T}{\Omega_T}-\frac{ga^2B_T}{\Omega_T}\right)|x|. \tag{62}$$

Therefore, the solution (56) becomes localized in x, i.e., $\lim_{x\to\pm\infty}\phi=0$, if

$$u_y=\frac{g}{\Omega_T} \tag{63}$$

and

$$c_1=-\frac{gB_Ta^2}{k_1\Omega_T}. \tag{64}$$

In that case the solution (56) can be written as a dimensionless function in the form

$$\Phi(x,y)\equiv\frac{\phi}{\frac{ga^2B_T}{\lambda_0\Omega_T}}=\ln[2\cosh(\lambda_0x)]-\frac{\lambda_0}{k_1}\ln\left[2\left(\cosh(k_1x)+\sqrt{1-\frac{1}{d_1^2}}\cos(k_1y)\right)\right]. \tag{65}$$

Finally, the corresponding ion concentration profile, which is necessary for such solution, is given by

$$n_{i0}(x)=\frac{1}{e}\exp\left[a^2\tanh(\lambda_0x)+\frac{gb_1^2a^2B_T}{\lambda_0\Omega_T}\ln[2\cosh(k_1x)]\right]. \tag{66}$$

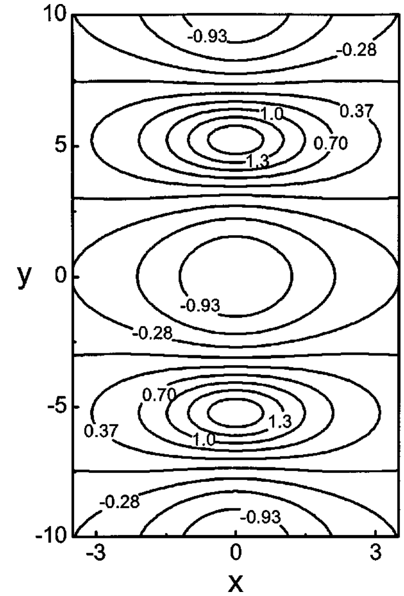

FIG. 3. The contour plot of the dimensionless potential of the vortex chain analytically given by Eq. (65). Here $\lambda_0=0.9$, $k_1=0.6$, and $d_1=1.6$.

Hence, the structure (65) is formed at the position in the x direction where the nonuniform ion drift velocity has the value g/Ω_T and its amplitude is dependent on this value and on the gradient of the velocity a^2. The same parameters enter the expressions for the electron and ion densities; however, the densities are also determined by another free parameter b so that the given conditions are not too strict. Similar to the tripolar vortex, the solution (65) is carried by the flow. The contour plot of the vortex chain is presented in Fig. 3. Here x is the direction of the gradients and the nonuniform ion drift is in the y direction. The structure is obtained for increasing profiles of the ion and electron concentrations given by Eqs. (59) and (66) and an almost flat profile of $Z_d(x)n_{d0}(x)$. The corresponding profiles for $n_{i0}(x)$, $n_{e0}(x)$, $Z_d(x)n_{d0}(x)$, and $B_0(x)$ are given in Figs. 4 and 5.

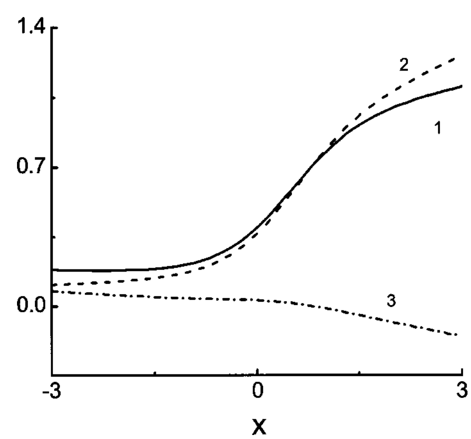

FIG. 4. The ion (curve 1) and electron (curve 2) density profiles given by Eqs. (59) and (66), respectively, for the potential from Fig. 3. The dash-dotted line (curve 3) is the profile of $Z_d(x)n_{d0}(x)$. Here $a^2=0.9$ and $gb^2B_Ta^2/\Omega_T=0.1$, and other parameters are the same as in Fig. 3.

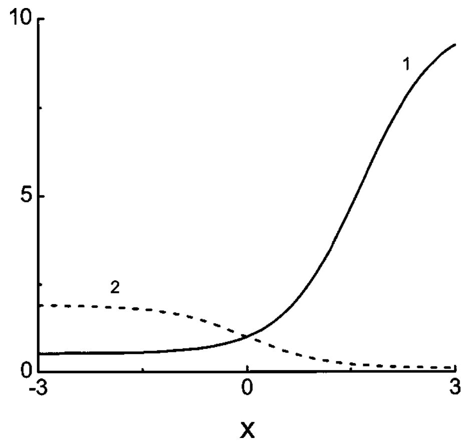

FIG. 5. The magnetic field (curve 1) and the velocity profile (curve 2) for the solution presented in Fig. 3.

VII. SUMMARY

We have studied linear and nonlinear regimes in the development of an interchange mode in a dusty plasma with stationary dust grains and with a spatially dependent charge on the grains. In addition, a reasonably realistic case with a spatially dependent magnetic field, and, consequently, with a nonuniform ion drift perpendicular to the magnetic field lines, is included in the study. The presence of dust, which enters the equations through the quasineutrality condition in the equilibrium only and its influence on the interchange mode have been studied earlier in Ref. [8], resulting in the conclusion that negatively charged grains stabilize the system against the interchange instability. However, the additional spatial dependence of the charge on the grains introduces new effects and the aforesaid stabilization can be violated. Similar effects follow from the action of the nonuniformity of the ion drift.

In the nonlinear domain we have derived corresponding nonlinear equations describing perturbations that propagate perpendicular to the magnetic field. They include several x-dependent functions which describe the equilibrium. The nonlinear equations comprise vector-product-type nonlinearities and can be integrated, resulting in two additional arbitrary functional forms (26) and (27) with the arguments $\phi(x,y)-B_T u_y x$ and $\phi(x,y)+\varphi(x)-B_T u_y x$. Therefore, various nonlinear solutions are possible, dependent on the choice of these equilibrium functions and functional forms. We have found three types of stationary solutions, in the form of tripolar and dipolar vortices and vortex chains. The dipolar vortex is shown to propagate with respect to the drifting plasma; two other structures are just carried by the plasma flow (drift). Both the tripole and chain are driven by the ion drift nonuniformity (i.e., by the nonuniformity of the magnetic field), but they are also strictly dependent on the equilibrium parameters describing densities.

The dust enters into the equations in the most simple way, i.e., through the quasineutrality condition (1) only. Yet it significantly influences the mode behavior in both the linear and nonlinear regimes. We have found analytical expressions for the equilibrium quantities which allow for the given structures; see Eqs. (39), (41), and (42) for the tripole, Eq. (51) for the dipole, and Eqs. (58), (59), and (66) (and the corresponding figures, Figs. 4 and 5) for the vortex chain. Therefore one might say that whenever the equilibrium is described by these expressions, one should expect the formation of the given coherent structures representing possible saturated states of the linearly unstable interchange mode. The conditions under which the structures are found look strict, yet there exist several integration constants and physical parameters that are chosen freely, implying higher probabilities for the formation of such structures. Also, the stationary structures follow from specific forms of the aforesaid functional forms which are chosen freely [see Eqs. (28) and (53)]. This implies a possible diversity in realistic situations, which could eventually be an obstacle for clear evidence of the present structures in some experimental studies or in observations in space plasmas.

It should be said that tripolar vortices have been observed in laboratory experiments with rotating fluids, where they develop from a perturbed monopole [26]. They are shown to be remarkably stable structures, surviving many rotations of the system. A structure of the same sort has been observed in nature as well [27] as a way of self-organization in sea water. In plasma systems, the first analytical solution of that type was predicted in Ref. [28]. As for the experimental verification of the tripolar vortex in plasmas, recently it has been obtained as a standing electrostatic global structure which develops due to nonlinear effects in a cylindrical laboratory plasma [29]. As for dipoles, after the early theoretical prediction [30], they have been observed on many occasions in plasmas and fluids [31–34]. A dipole reported in Ref. [34] is observed in the core of a galaxy, i.e., in a self-gravitating medium; it is a huge structure with dimensions measured in light years. Also, chains of vortices have been observed so far in various situations in fluids [35] and plasmas [36]. Consequently the solutions found in the present study are realistic as they have been predicted and observed elsewhere; we have found rather precise conditions under which they may appear, and we believe that the present study may be used as a solid basis for some experimental or observational searches for such structures in nonuniform dusty or nondusty plasmas.

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